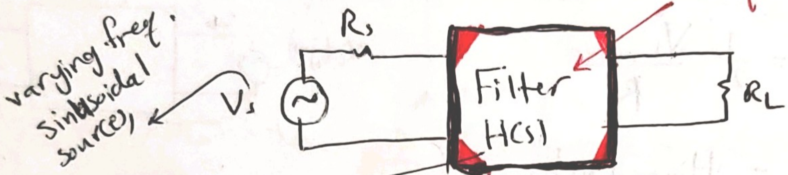
Introduction to Frequency Selective Circuits:-

[Filters]

- analoge Filter
  - Passive Filter
    - low-pass filter.
    - high-pass filter.
    - bandpass filter.
    - bandreject filter.
  - Active filter
- digital filter



Frequency-selective circuits.

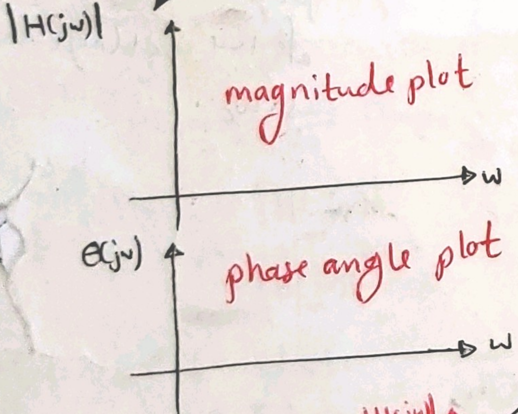
$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

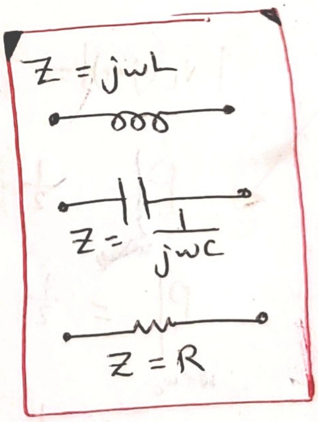
⟹ input ⟹ output

Frequency Response

Analysis

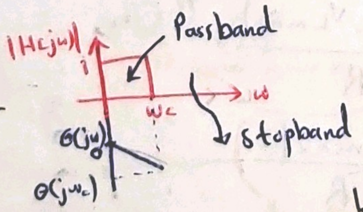
- Qualitative Analysis نوعي [1]
- Quantitative Analysis كمي [2]



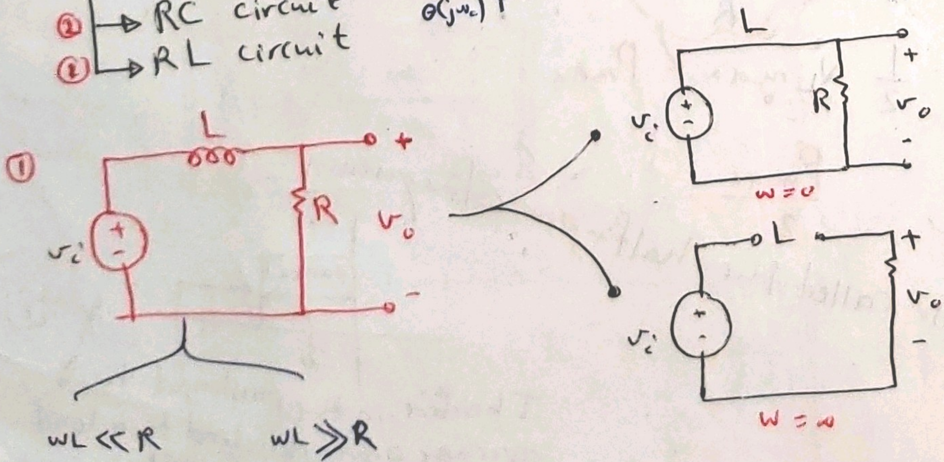


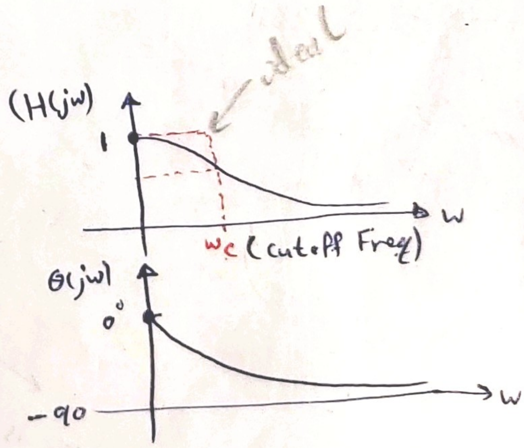
[1]

Low-Pass Filter



① → RC circuit
② → RL circuit





Defining the Cutoff Freq :-

Cutoff freq : is the frequency for which the transfer function magnitude is decreased by the factor $\frac{1}{\sqrt{2}}$ from its maximum value.

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

$|H(j\omega)|_{max}$

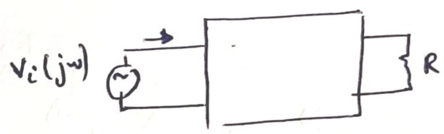
max magnitude of the transfer function.

amp of the voltage drop across the load

(average power delivered by any circuit to a load) $P = \frac{1}{2} \frac{V_L^2}{R}$

2 notes

1) $P_{max} = \frac{1}{2} \frac{V_{Lmax}^2}{R}$



2) $V_{Lmax} = H_{max} |V_i|$ ***

Now consider what happens to the average power when the freq of the voltage source is ω_c.

at ω_c

3) $|V_L(j\omega_c)| = \underbrace{|H(j\omega_c)|}\, |V_i|$

$$= \frac{1}{\sqrt{2}} \underbrace{H_{max} |V_i|}$$

$$|V_L(j\omega_c)| = \frac{1}{\sqrt{2}} V_{Lmax}$$

$$P = \frac{1}{2} \frac{V_L^2}{R}$$

$$P\Big|_{\omega=\omega_c} = \frac{1}{2} \frac{|V_L^2(j\omega_c)|}{R}$$

$$= \frac{1}{2} \frac{\left(\frac{1}{\sqrt{2}} V_{Lmax}\right)^2}{R}$$

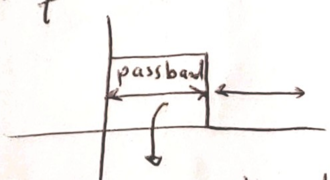
$$= \frac{1}{2} \frac{V_{Lmax}^2/2}{R}$$

P_{max}

$$= \frac{1}{2} \cdot$$

$$= \frac{P_{max}}{2}$$

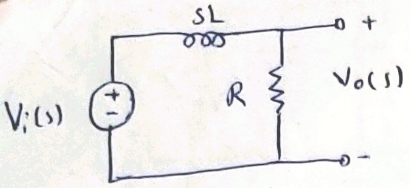
ω_c is also called the half-power frequency



2

Therefore, in the passband the average power delivered to a load is at least 50 % of the max avg.

⇒ Low-Pass Filter :



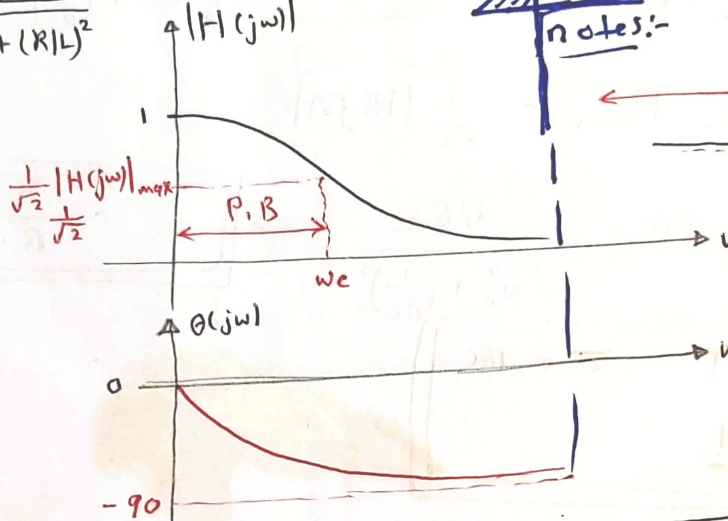
$$H(s) = \frac{R}{R+sL}$$

$$\Rightarrow \quad H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R/L}{s + R/L}$$

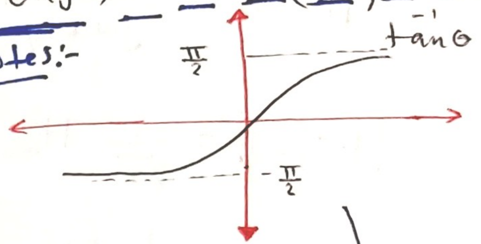
$$H(j\omega) = \frac{R/L}{j\omega + R/L}$$

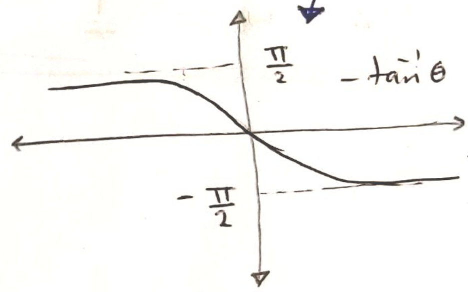
$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + (R/L)^2}}$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$



notes:-





$$\theta = -\tan^{-1}(\omega\tau)$$

Cutoff frequency

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max} \quad \text{at } \omega = 0$$

$$H_{max} = |H(j\omega)|_{\omega=0}$$

$$= |H(j0)|$$

$$\Rightarrow |H(j\omega_c)| = \frac{1}{\sqrt{2}}|1| = \frac{R/L}{\sqrt{\omega_c^2 + \left(\frac{R}{L}\right)^2}}$$

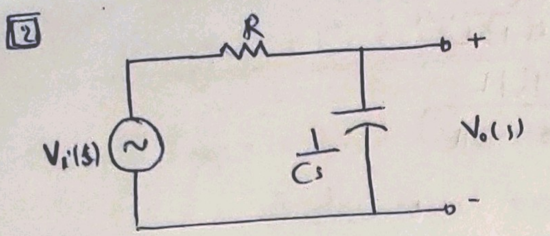
$$\therefore \Rightarrow \omega_c = \frac{R}{L}$$

$$\boxed{H(s) = \frac{\omega_c}{s+\omega_c}}$$

for RL circuit $\tau = \frac{L}{R}$

$$\therefore \omega_c = \frac{1}{\tau}$$

2



(a) $H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/Cs}{R + \frac{1}{Cs}} = \frac{1}{RCs + 1} = \frac{1/RC}{s + 1/RC}$

$H(j\omega) = \frac{1/RC}{j\omega + 1/RC} \Rightarrow |H(j\omega)| = \frac{1/RC}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$, $\theta(j\omega) = 0 - \tan^{-1}(\omega RC)$
$= -\tan^{-1}(\omega\tau)$

$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max} = \frac{1}{\sqrt{2}} |H(j0)|$

$= \frac{1}{\sqrt{2}}(1) = \frac{1/RC}{\sqrt{\omega_c^2 + \left(\frac{1}{RC}\right)^2}} \Rightarrow \boxed{\omega_c = \frac{1}{RC}}$

For RC CKT $\tau = RC$

$\therefore \omega_c = \frac{1}{\tau}$

$$\boxed{H(s) = \frac{\omega_c}{s + \cancel{1/RC}\ \omega_c}}$$

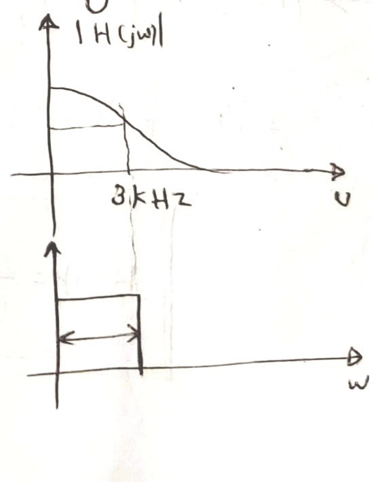
* Choose values for R, C (First) that will yield a low-pass filter with a cutoff frequency of 3 KHz.

$C = 1\ \mu F$

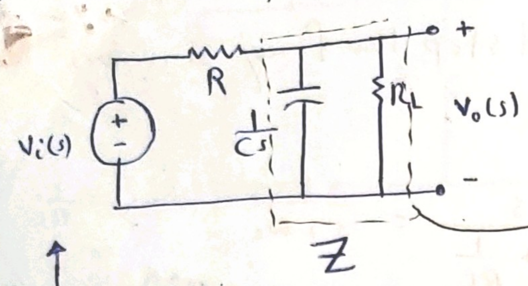
$\frac{1}{RC} = \omega_c = 2\pi(3\times10^3) = \frac{1}{R*1*10^{-6}}$

$R = \frac{1}{10^{-6} * 2\pi * 3 * 10^3} = 53\ \Omega$

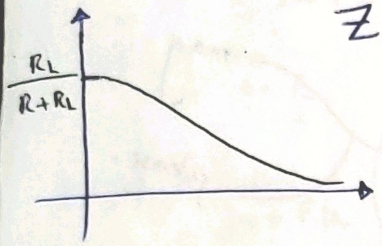
?! Design



» Loaded Low-pass filter :-



$$\frac{R_L(1/sC)}{R_L + \frac{1}{sC}} = \frac{R_L}{R_L C s + 1}$$



ⓐ
$$H(s) = \frac{\frac{R_L}{R_L C s + 1}}{\frac{R_L}{R_L C s + 1} + R}$$

$$= \frac{R_L}{\cancel{R_L C s + 1}} \cdot \frac{\cancel{R_L C s + 1}}{R_L + R(R_L C s + 1)}$$

$$= \frac{R_L}{R R_L C s + R + R_L}$$

$$= \frac{\frac{R_L}{R R_L C}}{s + \frac{R + R_L}{R R_L C}} = \frac{1/RC}{s + \frac{(R + R_L)}{R R_L C}}$$

$$\boxed{H(s) = \frac{1/RC}{s + \frac{R + R_L}{R R_L C}}}$$

$$|H(j\omega)| = \frac{1/RC}{\sqrt{\omega^2 + [(R+R_L)/R R_L C]^2}}$$

$\omega = 0$

$$|H(j\omega)|_{max} = \frac{R_L}{R + R_L}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\frac{R_L}{R + R_L}\right] = \sqrt{\omega_c}$$

$$\therefore \omega_c = \frac{R + R_L}{R R_L C} = \frac{1}{R_{eq} C} = \frac{1}{\tau}$$

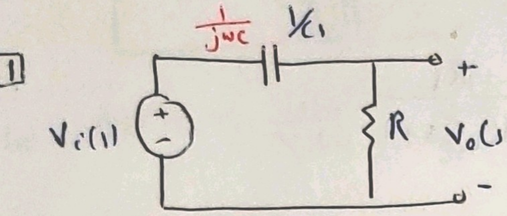
$$R_{eq} = (R \parallel R_L)$$

$$\tau = R_{eq} C$$

High-Pass Filters:

Are used to pass the high freq sinwaves and stop low freq sinwaves

[1]



$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{R}{R + \frac{1}{Cs}} = \frac{RCs}{RCs+1} = \frac{s}{s + \frac{1}{RC}} \approx \frac{jw}{jw + \frac{1}{RC}}$$

$$= \frac{s}{s + \frac{1}{RC}} = \frac{s}{s + w_c} \qquad w_c = \frac{1}{RC} = \frac{1}{\tau}$$

mag

phase.

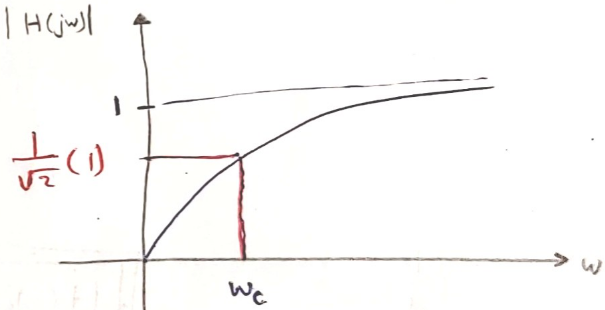
mag:

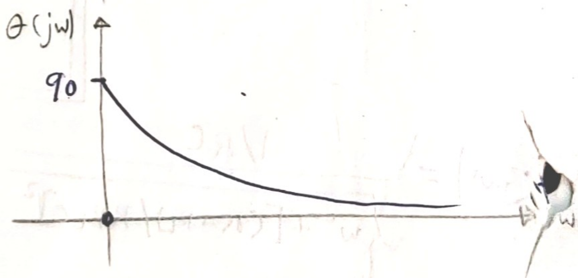
$$|H(jw)| = \frac{w}{\sqrt{w^2 + \left(\frac{1}{RC}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{1}{RCw}\right)^2}}$$

$w = 0 \Rightarrow |\text{mag}| = 0$

$w = \infty \Rightarrow |\text{mag}| = 1$

$$\begin{bmatrix} w\uparrow \\ (\;)\downarrow \\ \sqrt{\;}\downarrow \\ |\;|\uparrow \end{bmatrix}$$

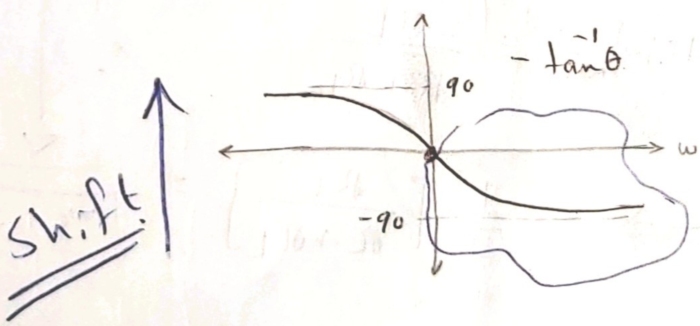




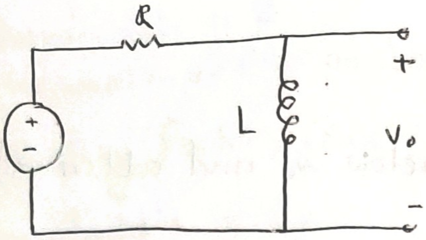
phase:

$$\theta(jw) = 90 - \tan^{-1}\left(\frac{w}{\frac{1}{RC}}\right)$$

$$= 90 - \tan^{-1} wRC$$



[2] :

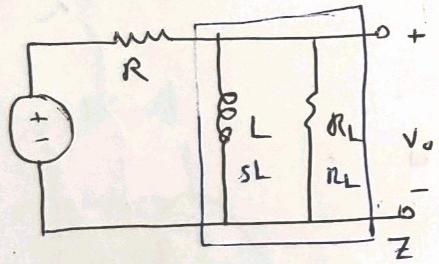


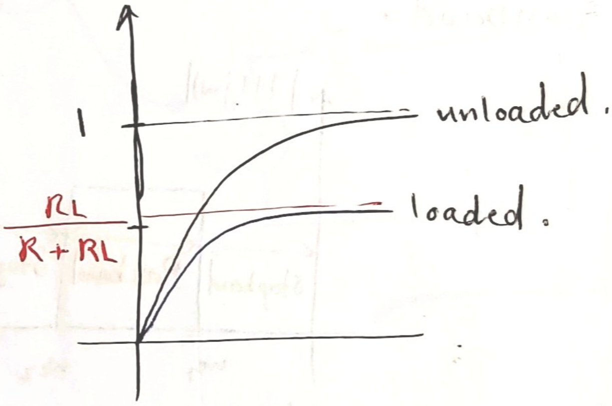
$$H(s) = \frac{s}{s + \frac{R}{L}}$$

$$H(j\omega) = \frac{j\omega}{j\omega + R/L}$$

$$\omega_c = \frac{R}{L} \Rightarrow \boxed{H(s) = \frac{s}{s + \omega_c}}$$

[3] Loaded High pass Filters :-





$$\boxed{\tau = \frac{L}{R_{eq}} , \quad R_{eq} = (R \,||\, R_L)}$$

$$\omega_c = \frac{1}{\tau} = R_{eq}/L$$

$$\boxed{\omega_c = \left(\frac{R R_L}{R + R_L}\right) / L}$$

$$H(s) = \frac{\left[\frac{R_L sL}{R_L + sL}\right]}{\left[R + \frac{R_L sL}{R_L + sL}\right]}$$

Can be written

$$= \frac{\left(\frac{R_L}{R + R_L}\right) s}{s + \left(\frac{R_L}{R + R_L}\right)\frac{R}{L}}$$

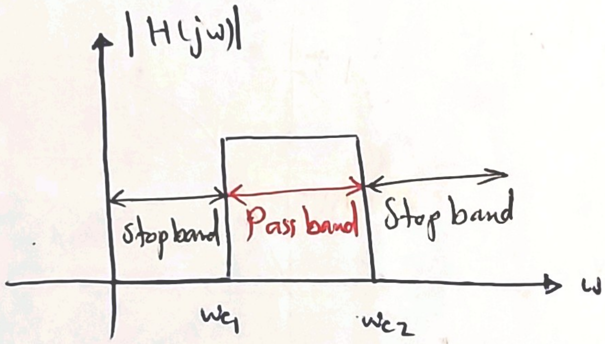
$$\boxed{H(s) = \frac{\left(\frac{R_L}{R + R_L}\right) s}{s + \omega_c}}$$

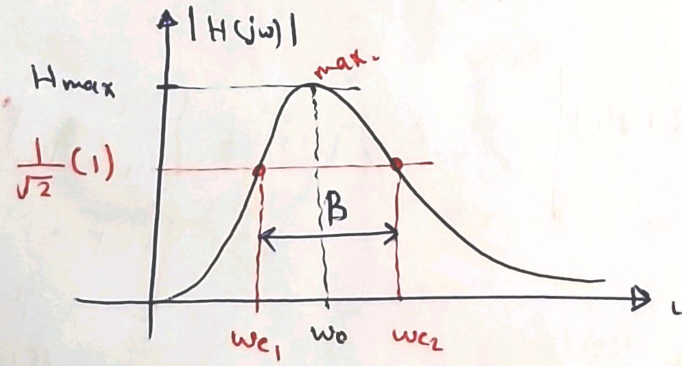
Bandpass Filters :

>> A low-pass filter passes voltages at frequencies below w_c and attenuates frequencies above w_c.

>> A high-pass filter passes voltages at frequencies above w_c, and attenuates voltages at frequencies below w_c.

>> A bandpass filter passes voltages at frequencies within the passband which is between w_{c1} and w_{c2}. It attenuates frequencies outside of the passband.





$\boxed{\Rightarrow \text{Second orde} \Rightarrow \text{RLC, CKT}}$

① $w_c \equiv$ cutoff frequency

② $w_o \equiv$ center frequency, w_o, defind as the frequency for which a circuit's transfer function is purely real.

$\equiv$ resonant frequency,

$\equiv$ geometric center of the passband.
mean

$\equiv \sqrt{w_{c1} w_{c2}}$

$H_{max} = |H(jw_o)|$ For bandpass Filters

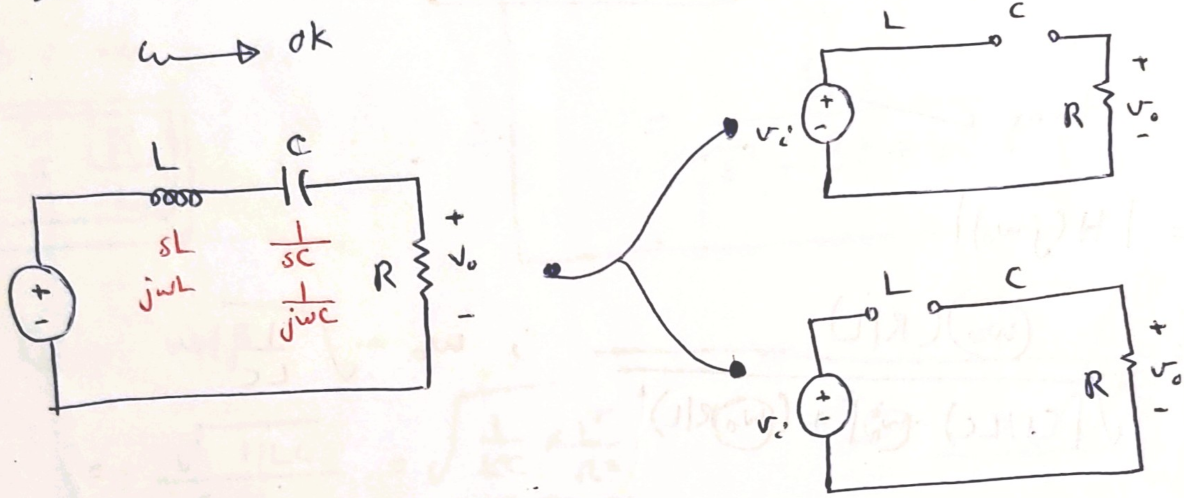
[3] $\beta \equiv$ Bandwidth $= \beta$
= the width of the passband.

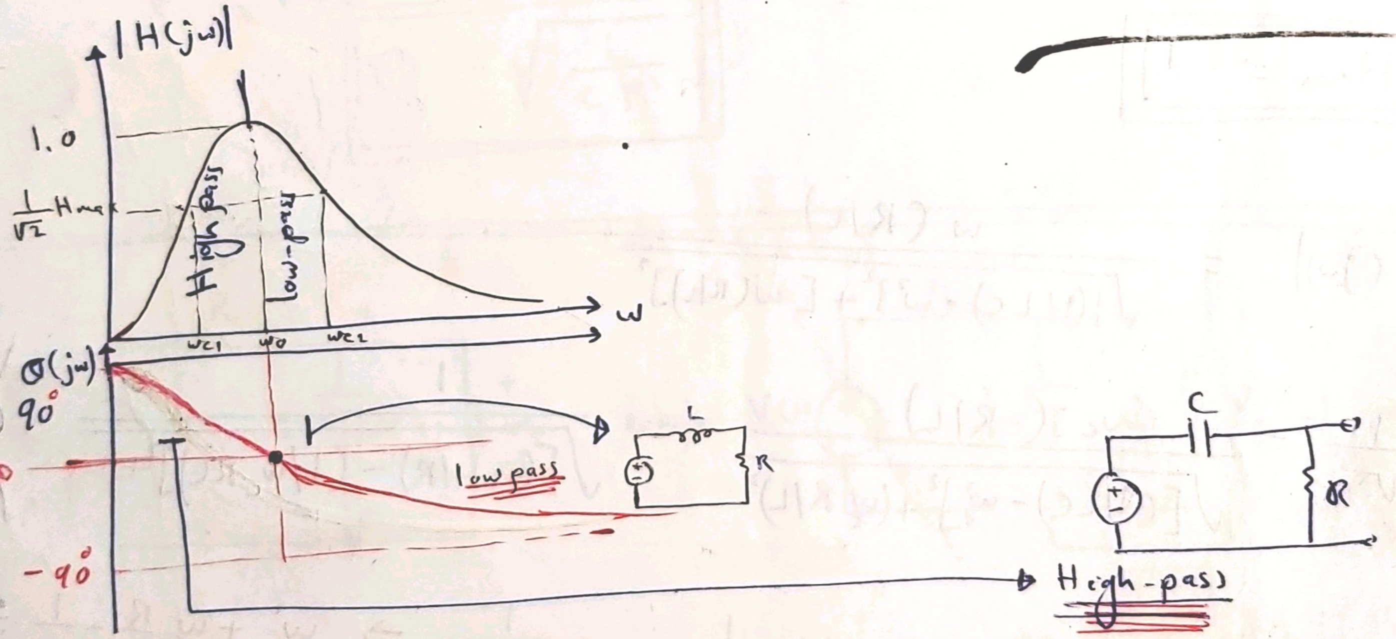
[4] $Q \equiv$ quality factor, which is the ratio of the center frequency to the bandwidth $\equiv \frac{\omega_0}{\beta}$

ω_{c1} ✓, ω_{c2} ✓, ω_0, β, Q

$\omega \longrightarrow$ ok

[1]





$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{s}{L} * R}{\frac{s}{L} * R + sL + \frac{1}{Cs}} = \frac{(R/L)s}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}} = \frac{R/L\, j\omega}{-\omega^2 + \frac{R}{L} j\omega + \frac{1}{LC}}$$

$$= \frac{R/L\, j\omega}{\left(\frac{1}{LC} - \omega^2\right) + \frac{R}{L}\omega j}$$

$$|H(j\omega)| = \frac{\omega(R/L)}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left[\omega\left(\frac{R}{L}\right)\right]^2}}$$

$$\theta(j\omega) = 90 - \tan^{-1}\left[\frac{\omega(R/L)}{(1/LC) - \omega^2}\right]$$

$\boxed{\omega_0, \ \omega_{c1}, \ \omega_{c2}, \beta, Q}$?!?

$\boxed{w_o}$

$$j w_o L + \frac{1}{j w_o C} = 0$$

$$j w_o L = \frac{j}{w_o C} \Longrightarrow w_o^2 LC = 1$$

$$\boxed{w_o = \sqrt{\frac{1}{LC}}}$$

$\boxed{w_{c1}, w_{c2}}$ H_{max} ✓

$$H_{max} = |H(j w_o)|$$

$$= \frac{w_o (R/L)}{\sqrt{[(1/LC) - w_o^2]^2 + (w_o R/L)^2}}, \quad w_o = \sqrt{\frac{1}{LC}}$$

$$\boxed{H_{max} = 1}$$

$$|H(j\omega)| = \frac{\omega (R/L)}{\sqrt{[(1/LC) - \omega^2]^2 + [\omega (R/L)]^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{w_c (R/L)}{\sqrt{[(1/LC) - w_c^2]^2 + (w_c R/L)^2}} = \frac{1}{\sqrt{[(w_c L/R) - (1/w_c RC)]^2 + 1}}$$

$$\pm 1 = w_c \frac{L}{R} - \frac{1}{w_c RC} \Rightarrow \underbrace{w_c^2 \pm w_c \frac{R}{L} - \frac{1}{CL} = 0}$$

$$\frac{\pm \frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} + 4(1)\left(\frac{1}{CL}\right)}}{2(1)}$$

$$w_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$w_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

2 (positive) ✓ ok

2 (negative) ✗

$\boxed{w_0}$:

$$w_0 = \sqrt{w_{c_1} \cdot w_{c_2}}$$

≡ geometric mean of the two cutoff frequencies.

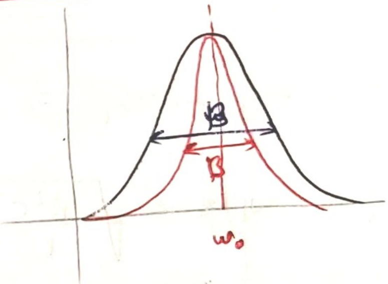
$$\boxed{w_0 = \sqrt{\frac{1}{LC}}}$$

$\boxed{\beta}$

$$\beta = w_{c_2} - w_{c_1}$$

$$\boxed{\beta = \frac{R}{L}}$$

$$\boxed{H(s) = \frac{(R/L)\ s}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}} = \frac{\beta s}{s^2 + \beta s + w_0^2}}$$



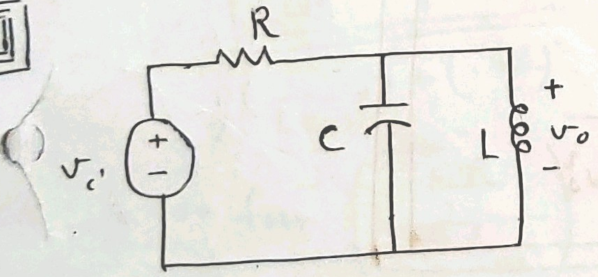
$\boxed{Q}$

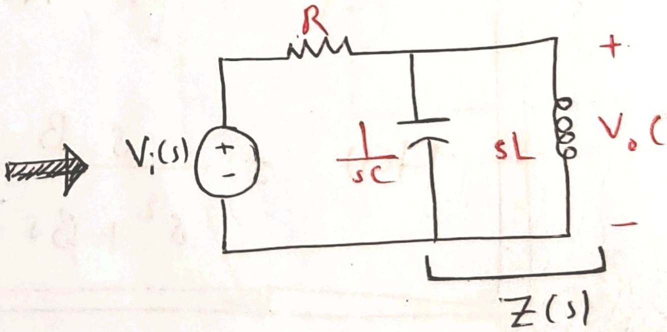
$$Q = w_0/\beta$$

$$= \frac{\sqrt{1/LC}}{R/L} = \sqrt{\frac{1}{LC} * \frac{L^2}{R^2}}$$

$$\boxed{Q = \sqrt{\frac{L}{CR^2}}}$$

$\boxed{2}$





$$\frac{\frac{1}{j\omega C} \cdot j\omega L}{j\omega L + \frac{1}{j\omega C}} = 0$$

$$Z(s) = \frac{\frac{1}{sC} \cdot sL}{\frac{1}{sC} + sL} = \frac{\frac{L}{C}}{sL + \frac{1}{sC}}$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{s/RC}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

$$H(jw) = \checkmark \qquad s = jw$$

$$|H(jw)| = \frac{w/RC}{\sqrt{\left(\frac{1}{LC} - w^2\right)^2 + \left(\frac{w}{RC}\right)^2}} = \frac{1}{\sqrt{1 + \left(wRC - \frac{1}{w\frac{L}{R}}\right)^2}}$$

$$* \frac{w}{RC}$$

$$\left(wRC - \frac{1}{w\frac{L}{R}}\right)^2$$

$$\left(w^2 - \frac{1}{LC}\right)^2$$

$$\left(\frac{1}{LC} - w^2\right)^2$$

$$\left(\frac{1}{LC} - w^2\right)^2 = 0$$

max

0

$$\boxed{\omega_0 = \sqrt{\frac{1}{LC}}}$$

$$H_{max} = |H(j\omega_0)| = 1$$

$$\frac{1}{\sqrt{2}} H_{max} = |H(j\omega_c)|$$

① ② } positive
③ ④ } 2 (+ve)

$$\omega_{c_1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_{c_2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = \frac{1}{RC}$$

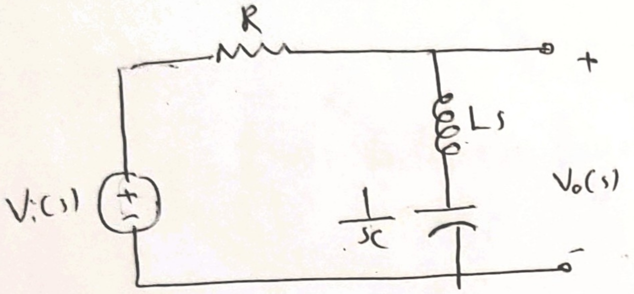
$$Q = \omega_0/\beta = \sqrt{\frac{R^2C}{L}}$$

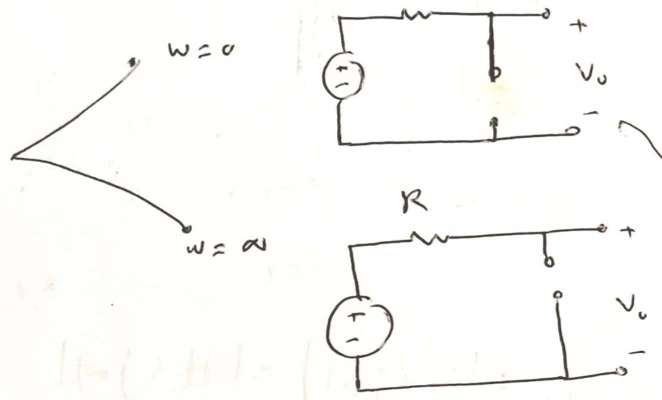
$$H(s) = \frac{s/RC}{s^2 + s/RC + \frac{1}{LC}}$$

$$H(s) = \frac{s\beta}{s^2 + \beta s + \omega_0^2}$$

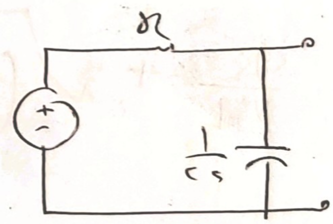
- Band reject Filter :
- Band Stop Filter

attenuates voltages at frequencies within the stopband, which is between w_{c_1} and w_{c_2}. It passes frequencies outside of the stopband.





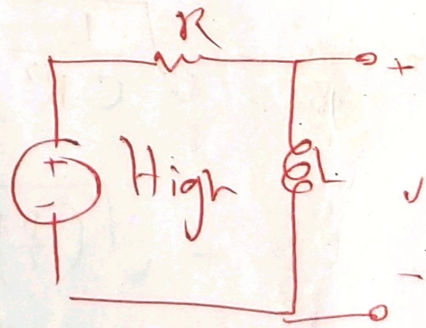
$$H(s) = \frac{\left(\frac{s}{L}\right)\left(sL + \frac{1}{sC}\right)}{\left(\frac{s}{L}\right)\left(R + sL + \frac{1}{sC}\right)} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{-w^2 + \frac{1}{LC}}{-w^2 + \frac{R}{L}jw + \frac{1}{LC}}$$

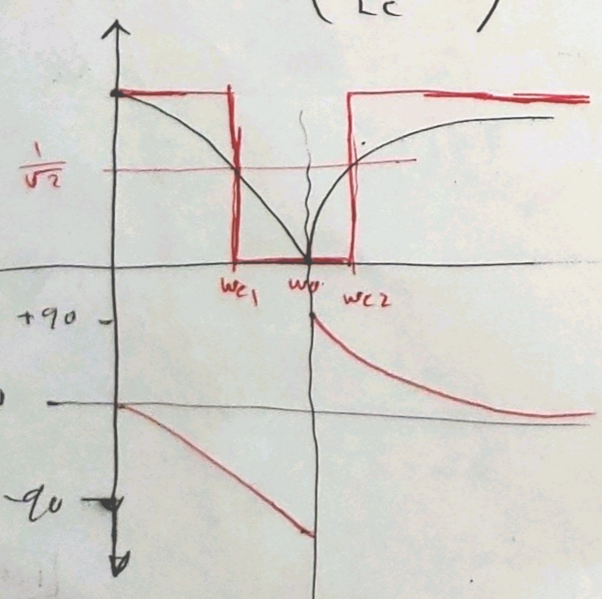


low-pass-filter

$$|H(jw)| = \frac{\left|\frac{1}{LC} - w^2\right|}{\sqrt{\left(\frac{1}{LC} - w^2\right)^2 + \left(\frac{wR}{L}\right)^2}}$$

$$\theta(jw) = 0 - \tan^{-1}\left(\frac{\frac{wR}{L}}{\frac{1}{LC} - w^2}\right)$$





$w_0 \equiv$ center frequency $\equiv$ reseonent freq

$$j w_0 L + \frac{1}{j w_0 C} = 0$$

$$w_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{|H(jw_0)| = 0 \quad (\text{min})}$$

$$\left(\frac{1}{\sqrt{2}} H_{max}\right) \longleftarrow \begin{matrix} w_{c_1} \\ w_{c_2} \end{matrix}$$

$$H_{max} = |H(j0)| = |H(j\infty)|$$

Here $H_{max} = 1$

$$w_{c_1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

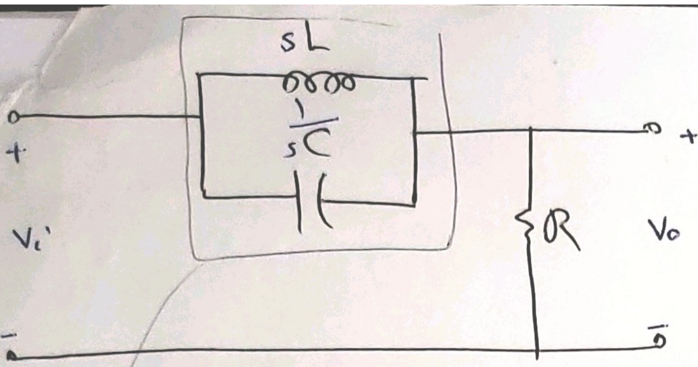
$$w_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\beta = \frac{R}{L}$$

$$Q = \frac{w_0}{\beta} = \sqrt{\frac{L}{CR^2}}$$

$$w_{c_1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + w_0^2}.$$

$$w_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + w_0^2}.$$



at $w = w_0$

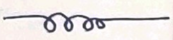
$sL \quad \frac{1}{sC} \Rightarrow \infty$

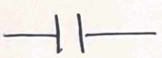
$s \sim \leftarrow 0$

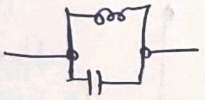
$\boxed{V_o = 0}$

qualitative analysis
نوعي

quantitative analysis
كمي

at $w = 0$  $\Rightarrow$ —— $\Rightarrow$ $v_o = v_i$

at $w = \infty$  $\Rightarrow$ —— $\Rightarrow$ $v_o = v_i$

at $w = w_0$  $\Rightarrow \infty \Rightarrow v_o = 0$

$$Z = \frac{sL(1/sC)}{sL + \frac{1}{sC}} = \frac{sL}{s^2LC + 1}$$

$$H(s) = \frac{V_o}{V_i} = \frac{R}{R + Z} = \frac{s^2 + 1/LC}{s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC}}$$

$$\boxed{H(s) = \frac{s^2 + w_0^2}{s^2 + \beta s + w_0^2}}$$

$$H(jw) = \frac{w_0^2 - w^2}{w_0^2 - w^2 + jw\beta}$$

$$w_0 = \frac{1}{\sqrt{LC}}$$

$H(jw) = 0$ when $w = w_0$

$$|H(jw)| = \frac{w_0^2 - w^2}{\sqrt{(w_0^2 - w^2)^2 + w^2\beta^2}} \Rightarrow \frac{1}{\sqrt{2}} = |H(jw)|$$

$$= \frac{1}{\sqrt{\frac{(w_0^2 - w^2)^2}{(w_0^2 - w^2)^2} + \frac{w^2\beta^2}{(w_0^2 - w^2)^2}}}$$

$$w^2\beta^2 = (w_0^2 - w^2)^2 \Rightarrow \pm w\beta = w_0^2 - w^2$$

$$w^2 \pm \beta w - w_0^2 = 0$$

$$w_{c_1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + w_0^2}$$

$$w_{c_2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + w_0^2}$$